# Foreign Exchange Hedging and Currency Invoicing: Part I Theory *

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**Abstract**

We study how firms jointly determine currency invoicing and foreign exchange (FX) hedging by incorporating forward contract transactions into a canonical currency-choice framework. We show that (i) in the presence of imperfect hedges and (ii) hedging and pricing decisions are interconnected, FX hedging costs *almost always* affect currency invoicing. This interaction operates through two distinct channels: First, firms manage exchange-rate risk through two margins—currency invoicing and FX hedging—and hedging costs shape the relative use of these two tools. Second, hedging costs affect firms' hedging positions, which in turn alter exchange-rate pass-through into prices and thereby affect optimal currency choice. Specifically, lower hedging costs in dollars facilitate more effective risk management for firms' dollar exposure, incentivizing them to invoice in dollars—an effect especially pronounced for firms with long dollar positions. Altogether, our results elucidate how dollar dominance in FX transactions reinforces its dominance in trade invoicing, highlighting a fundamental policy tension between the internationalization of domestic currencies and the deepening of FX hedging markets.

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# 1 Introduction

The U.S. dollar occupies a central position in the international monetary system (Gourinchas et al., 2019; Gopinath and Itskhoki, 2022). For global firms, this dominance is especially salient across two distinct yet interconnected margins: on the *financial side*, through the currency composition of foreign-exchange (FX) markets, and on the *real side*, through the choice of currency used to invoice international trade. While the dollar maintains a preeminent share in both margins, the mechanisms that connect them remain an open question. This paper provides a general framework and finds that violations of Modigliani and Miller (1958) drive this link and reshape the optimal invoicing decision.

Understanding this connection is critical for several reasons. First, firms actively manage exchange-rate exposure through FX derivatives (Adams and Verdelhan, 2022; Alfaro et al., 2025; Hacioglu Hoke et al., 2026) and currency invoicing (Gopinath et al., 2010; Amiti et al., 2022). Second, currency invoicing remains a central pillar of international macrofinance (Gopinath et al., 2020). It serves as the primary mechanism governing exchange-rate pass-through, thereby determining how exchange rate fluctuations propagate to domestic real economies (Atkeson and Burstein, 2008a; Auer et al., 2021). Lastly, the question carries significant policy weight. Emerging market economies are often heavily dollarized (Eichengreen et al., 2005) and exposed to severe currency mismatches (Salomao and Varela, 2022), while in the U.S., policymakers continue to debate the future of dollar dominance.

Despite its central importance, the existing literature faces a significant limitation: we still lack a general theoretical framework that *jointly* studies firms' currency invoicing decisions and their financial risk management. We aim to fill this gap.

In this paper, we propose that a firm's currency invoicing decision and its financial risk management are two sides of the same coin. Consider a Colombian firm export goods to a US market. It has to *preset* three key decisions before exchange-rate shocks are realized: its nominal price, its financial hedging position, and its invoicing currency. In a frictionless world, the firm would follow "state-contingent" benchmarks—the optimal price and hedging positions it would choose if it could react to shocks flexibly. However, nominal and financial rigidities induce a value loss whenever realized outcomes deviate from these ideal cases. Maximizing expected firm value is thus equivalent to minimizing the *combined tracking error* of both the nominal price and the financial hedge relative to their respective state-contingent benchmarks. We define this fundamental tension as the *pricing-hedging trade-off*: an invoicing choice is optimal only if it minimizes the *joint* distortion across both the real and financial margins.

This perspective generalizes the canonical currency choice framework in Engel (2006), where the only risk-management tool available to the firm is the choice of invoicing currency. In that world, the firm effectively "pre-loads" exchange-rate risk into its pricing rule to stabilize the price in the chosen unit of account—a *pure* indexing problem. While this indexing intuition is powerful, our framework shows it is incomplete when a second margin, financial hedging, is active. In our setting, currency invoicing evolves *beyond* a standalone indexing problem; because exchange-rate risk is partially mitigated through financial hedging, the optimal invoicing is determined by how the chosen currency complements the firm's broader risk-management strategy.

Our main theoretical result in Section 2 is as follows: (i) in the absence of perfect financial hedges for currency risks and (ii) when risk management and real decisions are tightly connected, foreign exchange (FX) hedging *almost always* affects currency choice. Notably, condition (ii) admits a natural interpretation through the lens of classic corporate finance: under a set of strong assumptions, financing decisions—including hedging—do not affect a firm's production decisions, such as pricing. This separation logic dates back at least since Fisher (1930) and remains the cornerstone of the Modigliani and Miller (1958) theorem (henceforth MM). Our framework thus bridges this classic corporate finance principle with the currency invoicing literature in international macroeconomics.

Our main theoretical result in Section 2 is as follows: when financial hedges for currency risk are imperfect, FX hedging *almost always* affects currency choice whenever financial and real decisions are non-separable. This non-separability is stronger than the usual statement that a breakdown of MM makes hedging relevant for firm value. For hedging to matter for invoicing, it is not enough that the marginal value of hedging be non-zero. Hedging must also affect the marginal value of pricing, so that financial and operational risk management interact. In our model, this mechanism is summarized by a non-zero cross-derivative between firm value with respect to preset prices and hedging positions. Once this term is non-zero, firms manage exchange-rate risk through two jointly determined instruments—currency invoicing and FX hedging—and the cost of using one instrument changes the optimal use of the other. This provides a precise route through which deviations from MM translate into currency choice.

This insight allows us to reconcile the recent debate regarding the theoretical "null result"—the condition under which FX hedging remains decoupled from currency invoicing. On the one hand, the canonical literature synthesized by Gopinath and Itskhoki (2022) suggests that if hedging is merely a "side bet," it should not shift the invoicing decision. From the lens of our model, this view is theoretically consistent: it represents the limiting case where MM holds and FX transactions function as a carry trade (Bruno and Shin, 2017) rather than as risk management integrated with production.

On the other hand, recent work by Xie (2025) provides a vital breakthrough by identifying how contracting frictions can break this decoupling. Our framework is naturally complementary to this line of inquiry: we formalize the foundational insight that financial and real margins are linked by showing that the underlying mechanism is significantly more general. Specifically, this link does not rely on one *particular* friction, such as sticky quantities, but emerges whenever corporate risk management is connected with real production decision.

Consequently, our approach provides a structural roadmap for future research to identify which firm characteristics connect hedging to production decisions. It also allows both theoretical and empirical work to more precisely trace how corporate risk management scales up to shape the international monetary system.

From this broader risk-management perspective, the theoretical "null result" appears considerably more fragile than assumed in the canonical currency choice literature. Indeed, *any* friction that connects pricing and hedging—whether originating from financial constraints (Froot et al., 1993; Rampini and Viswanathan, 2010), asymmetric information (DeMarzo and Duffie, 1995), or managerial incentives (Smith and Stulz, 1985)—necessarily forces a re-evaluation of the firm's optimal invoicing strategy. This non-separability is well-supported by evidence documenting that FX hedging is a widespread activity (Alfaro et al., 2021; Hacioglu Hoke et al., 2026) that serves as a substitute for operational hedges (Hoberg and Moon, 2017) and co-moves with real exposure (Adams and Verdelhan, 2022; Liao and Zhang, 2025). Since fluctuations in hedging costs have documented real effects on firm behavior (Jung, 2025), a model of joint optimality better reflects corporate reality than the traditional separation benchmark.

We illustrate the power of our theoretical approach through two concrete structural examples. The first example in Section 3 builds on the seminal model of corporate risk management in Froot, Scharfstein and Stein (1993). This analysis achieves two goals: (i) it confirms that even in this minimal risk-management setting, a firm's hedging and pricing decisions are intrinsically interconnected, demonstrating that the pricing-hedging trade-off is a fundamental consequence of financial frictions rather than a modeling artifact; and (ii) it characterizes how this interaction grows as deviations from the flexible benchmark grow —for instance, when the cost of external equity is high or collateral haircuts on forward contracts are large. These results imply that it is precisely the firms furthest from the MM benchmark that are most likely to switch their invoicing currency when hedging costs fluctuate.

Our second example in Section 4 embeds literal FX forward contract transaction into a model of cash-flow hedging to uncover a second, distinct motive for currency choice. To our best knowledge, this mechanism has not been previously identified in the literature: FX risk management *endogenously* shifts the exchange-rate pass-through into desired prices. Specifically, we find that lower hedging costs in a specific currency (e.g., the dollar) encourage pricing in that currency, as the downside risk of depreciation is effectively hedged away. This effect is particularly pronounced for firms longing large net-dollar exposures, such as those with low foreign-input costs (Amiti et al., 2022). These predictions are directly testable in firm-level data. They also provide a theoretical foundation for how the availability of deep financial markets in a specific currency can cement its status as a dominant invoicing unit.

Altogether, our findings suggest that dollar dominance in financial transactions and in trade invoicing are mutually reinforcing. As demonstrated by our examples, these interactions are particularly potent for firms with large net-dollar exposures, those facing elevated hedging costs, or those constrained by frictions in equity issuance. This synergy carries two major policy implications. First, for emerging markets, policymakers face a fundamental trade-off between the internationalization of their domestic currency and the necessity of developing deep FX markets to mitigate dollar-denominated risks. Second, our results unveil how the unique role of the U.S. in dollar intermediation serves as a structural pillar that sustains the currency's dominance in international trade. By reducing the costs of risk management, deep financial markets in a specific currency endogenously lower the barriers to its use as an invoicing unit, creating a self-reinforcing cycle of global dominance.

**Related Literature** First, the paper relates to the literature on exchange-rate pass-through and currency invoicing. A large body of work studies how nominal rigidities, strategic complementarities, and invoicing choices shape the transmission of exchange rates into international prices, reviewed by Burstein and Gopinath (2014).[1] While recent work has begun to underscore the role of financial factors in these questions-including currency financing in working capital (Bahaj and Reis, 2020; Cristoforoni and Errico, 2025) on currency invoicing and the role of foreign currency debt on exchange-rate pass-through (Mereb and Ospina-Tejeiro, 2024)—we emphasize a distinct channel. By adding an endogenous hedging choice to the framework of Engel (2006), we find that FX hedging almost always influences currency choice as long as some choices are preset.[2] By centering the interaction

[1]This literature has evolved from studying standard producer-currency pricing (PCP) and local-currency pricing (LCP) (Obstfeld and Rogoff, 2000; Burstein et al., 2005; Bems and Di Giovanni, 2016; Cravino, 2017) toward the dominant-currency paradigm (DCP) (Gopinath and Rigobon, 2008; Amiti et al., 2019; Itskhoki and Mukhin, 2021). A key focus within this evolution is the role of strategic complementarities and variable markups in shaping pass-through (Atkeson and Burstein, 2008b; Amiti et al., 2014; Fitzgerald and Haller, 2014; Auer et al., 2021, 2024).

[2]The currency invoicing literature has progressed from early empirical regularities (Grassman, 1976) to the new open economy model beyond Mundell and Fleming's framework, for example (Friberg, 1998; Devereux and Engel, 2003; Bacchetta and Van Wincoop, 2005; Engel, 2006). More recent work has shifted focus toward

between risk management and pricing, we provide a new mechanism for currency choice that complements existing financial and real explanations.

Meanwhile, our framework is most closely related to Xie (2025), who also emphasizes the role of financial hedging in firms' currency choice. In his framework, the connection between hedging and invoicing is driven by a contracting friction in international trade, whereby invoicing currency serves to allocate exchange-rate risk across trading partners. We take a step back and ask under what sufficient conditions financial hedging is irrelevant for currency choice in the first place. This perspective leads us to focus on a *within-firm* margin, namely, the interaction between the firm's hedging and pricing decisions. We show that FX hedging affects currency choice whenever financial and real decisions are non-separable, that is, whenever the MM irrelevance benchmark fails. Taken together, both our paper and Xie (2025) suggest that financial hedging does affect currency choice, but through different channels. This broader perspective allows us to categorize which specific types of MM violations are most theoretically and empirically relevant—a taxonomy we develop in our ongoing and future research.

Second, this paper contributes to the dominant currency paradigm by highlighting the deep connection between two salient manifestations of dollar dominance: its central role in FX hedging markets and its dominant role in trade invoicing. As such, we add to the literature studying why a small set of currencies, particularly the U.S. dollar, plays a disproportionate role in the international monetary system (Gopinath et al., 2020). While existing work emphasizes several underlying functions of dominant currencies—including their role as a unit of account, a store of value, and a medium of exchange[3]—our approach belongs to a more recent strand of literature that highlights the mutually reinforcing interactions across these dimensions (Gopinath and Stein, 2021; Bahaj and Reis, 2020; Keller, 2024). Crucially, our modeling framework allows us to identify the micro-level heterogeneity underlying these macro trends. By pinpointing specific MM deviations, we elucidate which firms' invoicing decisions are most sensitive to fluctuations in hedging costs. This provides a clear set of testable predictions for how firm-level financial frictions systematically shape aggregate currency choice in international trade.

Lastly, the paper relates to the literature on corporate risk management and FX hedging. We build on extensive theoretical and empirical observations in this field to link corporate risk management to a specific real-side margin: currency invoicing. Specifically,

the rise of dominant currencies (Gopinath et al., 2010; Goldberg and Tille, 2016; Ito and Chinn, 2014; Corsetti et al., 2020), as well as the emerging importance of financial constraints and risk management (Drenik et al., 2022; Xie, 2025).

[3] See, for example, Amiti et al. (2022); Gopinath and Itskhoki (2022); Mukhin (2022) on the unit of account; Caballero et al. (2008); Maggiori (2017); Farhi and Maggiori (2018); Brunnermeier et al. (2024); Clayton et al. (2025) on the store of value; and Matsuyama et al. (1993) on the medium of exchange.

we highlight that the essence of firm risk management is to replicate a “first-best” state-contingent plan with limited instruments, such as the choice of currency. Which margins a firm adjusts—whether via financial instruments or real activities—depends entirely on the nature of the underlying frictions. Our framework demonstrates that when financial hedging markets are more frictional, firms are forced to rely more heavily on real-side adjustments, such as changing their pricing and invoicing decisions.

In this sense, our work extends the logic of Alfaro, Calani and Varela (2021), but with a crucial distinction in the mechanism. While they emphasize real adjustments through "natural hedging"—the difficult and often constrained process of matching payables and receivables in the same currency—we focus on the adjustment of pricing and invoicing. Because these nominal margins are a widespread and fundamental feature of international trade, our framework suggests that the link between financial frictions and real firm behavior can be even more pervasive than previously documented. By providing this microfoundation, we show that what is typically viewed as a private corporate risk management decision has significant macro implications for the stability and structure of the international monetary system.

## 2 General Model

In this section, we develop a streamlined framework to elucidate the interaction between foreign exchange (FX) hedging and invoicing currency choice. We deliberately adopt a generalized profit function as our primitive, which is determined by the firm's preset price and a singular hedging instrument. This level of abstraction is advantageous for two primary reasons.

First, it constitutes a minimal departure from the classical currency choice literature as in Engel (2006). While the firm retains the optimal selection of price and invoicing currency, our framework introduces only one additional endogenous variable: the hedging position. This parsimony ensures that the mechanisms driving our results remain tractable.

Second, this approach allows for a transparent characterization of how our findings depend on the validity of the MM benchmark. Under conditions where pricing and hedging decisions are fully separable, the hedging strategy remains neutral to the currency choice. Conversely, deviations from the MM theorem establish a direct, non-trivial link between these two corporate decisions, which we explore in the following analysis.

## 2.1 Set Up

**Environment** We consider a Colombian firm that produces cherries for export to the U.S. market. The firm makes three strategic decisions: (i) the invoicing currency, denoted by $\iota$, which can be either the producer currency—the Colombian peso (COP)—or the U.S. dollar (USD)[4]; (ii) the price of its goods; and (iii) its hedging position.

The model consists of two periods. At date $t$, prior to the resolution of uncertainty, the firm chooses its hedging position $\overline{H}$, its invoicing currency $\iota$, and its price $\overline{P}$. At date $t+1$, the exchange rate $S_{t+1}$ is realized and profits are determined. The exchange rate is treated as exogenous and represents the sole source of uncertainty in the model. We define the exchange rate $S$ as pesos per dollar, such that an increase in $S_{t+1}$ corresponds to a dollar appreciation against the peso. Throughout the analysis, lowercase letters denote logarithms (e.g., $x = \log X$), and variables marked with an asterisk denote prices expressed in pesos.

**Firm's Objective (Preset Plan)** The firm's optimization problem can be reformulated in two stages. In the first stage, we fix the invoicing regime – dollar pricing (USD invoicing, or USDI) versus peso pricing (COP invoicing, or COPI) –with $\iota \in \{\text{USDI}, \text{COPI}\}$.

Under a given regime, the firm selects a preset price $\overline{p}^{\iota}_{i,t}$ and a singular hedging position $\overline{H}^{\iota}_{i,t}$ at date $t$ to maximize the expected value of the firm, $\Pi_{i,t}$, prior to the realization of $s_{t+1}$. The solution for each regime is defined as:

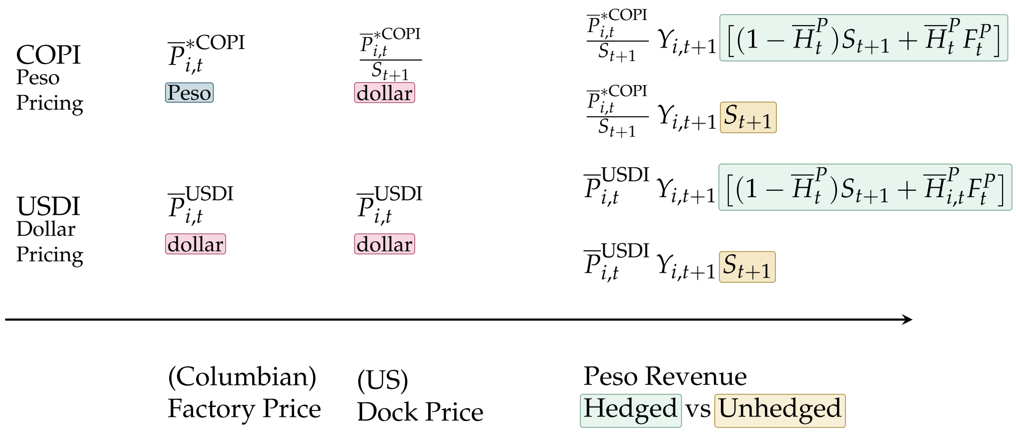
$$\left(\overline{p}^{\iota}_{i,t}, \overline{H}^{\iota}_{i,t}\right) \equiv \arg\max_{p_{i,t}, H_{i,t}} \mathbb{E}_t \left[\Pi_i(p_{i,t}, H_{i,t}; s_{t+1})\right] \text{ for } \iota \in \{\text{USDI}, \text{COPI}\}. \tag{1}$$

The invoicing regime dictates which nominal price is fixed at date $t$ and, consequently, how the U.S. dollar "dock price" faced by U.S. buyers evolves relative to the exchange rate at $t+1$. Figure 1 summarizes this notation, distinguishing between the Colombian factory-gate price and the U.S. dock price under both regimes.[5] Under dollar-pricing, the firm presets the Colombian factory-gate price $\overline{P}^{\text{USDI}}_{i,t}$ in U.S. dollars; thus, the U.S. dock price at $t+1$ remains $\overline{P}^{\text{USDI}}_{i,t}$ regardless of $S_{t+1}$. Under peso-pricing, the firm presets the Colombian factory-gate price $\overline{P}^{*\text{COPI}}_{i,t}$ in pesos; as a result, the U.S. dock price at $t+1$ is $\overline{P}^{*\text{COPI}}_{i,t}/S_{t+1}$ and fluctuates one-for-one with the realized exchange rate. The effective $p^{\iota}_{i,t}$ entering the profit

[4]To maintain analytical simplicity, we do not distinguish between dollar pricing (local currency pricing) and dominant currency pricing; however, this distinction can be incorporated into our framework in a straightforward manner. One can also think of the "dollar" as any foreign currency without loss of generality.

[5]In the benchmark model, we leave the firm's revenue and cost functions unspecified to maintain generality.

Figure 1: Notation for Pricing




function is the log price at the U.S. dock in U.S. dollars.

In the second stage, having determined the optimal price $\overline{p}^{\iota}_{i,t}$ and hedge $\overline{H}^{\iota}_{i,t}$ for each regime, the firm compares the resulting expected profits and selects the invoicing currency $\iota_{i,t}$ that yields the highest value:

$$\iota_{i,t} = \arg \max_{\iota \in \{\text{USDI,COPI}\}} \left\{ \mathbb{E}_t \left[ \Pi_i(\overline{p}^{\iota}_{i,t}, \overline{H}^{\iota}_{i,t}; s_{t+1}) \right] \right\}. \tag{2}$$

We refer to the complete solution $(\overline{p}^{\iota}_{i,t}, \overline{H}^{\iota}_{i,t}, \iota_{i,t})$ as the "preset plan," as it is determined optimally before the exchange-rate shock is realized.

**Remark 1** (Functional Form of $\Pi_i(p, H)$)**.** First, for tractability, we assume that the firm's value function is concave in both its pricing ($p_{i,t}$) and hedging ($H_{i,t}$) decisions, ensuring a unique interior solution. For pricing, this is a natural assumption and is satisfied in a wide range of monopolistic pricing models in the New Keynesian literature. For hedging, an interior solution can be obtained by assuming that the firm is sufficiently risk averse, so that the value function is concave in hedging even though profits are convex in input prices. For now, we remain agnostic about the precise source of the firm's effective risk aversion. [6] Section 3 and Section 4 provide two concrete examples for the function form.

Moreover, the MM insight is captured by the interaction term $\Pi_{pH}(p, H)$. If $\Pi_{pH}(p, H) = 0$, the hedging position $\overline{H}_{i,t}$ does not affect the marginal value of price adjustments, rendering hedging irrelevant to the currency choice $\iota_{i,t}$. Conversely, when $\Pi_{pH}(p, H) \neq 0$, the

[6] It may stem from costly external finance (Froot et al., 1993; Adams and Verdelhan, 2022) and collateral constraints (Rampini and Viswanathan, 2010); from managerial or owner non-diversification, as in classic theories of corporate hedging (Stulz, 1984; Smith and Stulz, 1985); from the fact that business losses directly threaten the owner's consumption, especially for small firms (Killeen, 2025); or from motives to smooth payouts, such as dividends (Hommel and Piquard, 2025).

pricing and hedging decisions become interdependent, allowing the hedging strategy to fundamentally influence the optimal invoicing regime.

Lastly, we assume that the functional form of $\Pi_i(\cdot)$ is invariant across invoicing regimes. This symmetry is standard in the literature (e.g., Gopinath et al., 2010) and makes our results conservative. For example, suppose that a one-dollar hedge offsets cash-flow risk more effectively under dollar invoicing than under peso invoicing. Then the firm would mechanically prefer dollar invoicing, not because of any endogenous interaction between pricing and hedging, but because the model assumes from the outset that hedging works better in that regime. By ruling out such built-in asymmetries, we show that the interaction between nominal rigidities and financial hedging alone is sufficient to shape the firm's optimal invoicing decision.

## 2.2 Solution

Solving the firm's optimization problem in its general form is analytically intractable. To obtain closed-form insights, we follow the established literature by employing a second-order Taylor approximation of the preset plan around a state-contingent plan, as seen in Engel (2006), Gopinath et al. (2010), and Amiti et al. (2022). This approximation allows for an analytical characterization of the interaction between uncertainty and corporate choice while preserving model tractability.

**State-Contingent Plans** The state-contingent plan addresses the following question: given a specific realization of the exchange rate $s_{t+1}$, what pair of price and hedging position would maximize the firm's value in that particular state? This benchmark identifies the firm's optimal allocation state-by-state, abstracting from the constraints of preset pricing.

This object is hypothetical. In our setting, it is inherently counterfactual to assume a firm selects its hedging position after the exchange rate is realized. We therefore introduce this "desired" allocation solely as an *analytical benchmark* to provide a convenient reference point for characterizing the actual preset allocation. [7] Specifically, for a given invoicing regime $\iota$, we define the desired price $\widetilde{p}^{\iota}(s_{t+1})$ and hedging position $\widetilde{H}^{\iota}(s_{t+1})$ as:

$$\left(\widetilde{p}^{\iota}(s_{t+1}),\, \widetilde{H}^{\iota}(s_{t+1})\right) \equiv \arg\max_{p,H}\ \Pi_i(p, H;\, s_{t+1})\,. \tag{3}$$

By defining these as functions of the realized exchange rate $s_{t+1}$, we characterize the choices under full flexibility. Because the profit function maintains the same functional form under

[7] Throughout the paper, we use state-contingent plan, desired allocation, and first-best benchmark interchangeably.

both invoicing regimes, the desired real allocation is identical across regimes, as summarized in the following lemma.

**Lemma 1** (Properties of the State-Contingent Plan)**.** The state-contingent plan satisfies:

$$\widetilde{p}^{\mathrm{USDI}}(s_{t+1}) = \widetilde{p}^{*\mathrm{COPI}}(s_{t+1}) - s_{t+1}, \quad \text{and} \quad \widetilde{H}^{\mathrm{USDI}}(s_{t+1}) = \widetilde{H}^{\mathrm{COPI}}(s_{t+1}). \tag{4}$$

Regarding pricing, this result implies that under full flexibility, the currency choice is merely a choice of the unit of account. For instance, if a firm intends to charge 1 USD in a specific state where the exchange rate is 3,800 COP/USD, this is economically equivalent to charging 3,800 COP. Since this logic holds for every realization of $s_{t+1}$, state-contingent prices across regimes differ only by the exchange rate adjustment, representing the same real allocation in different nominal units.

Furthermore, the desired hedging position $\widetilde{H}(s_{t+1})$ is invariant to the invoicing regime. This follows from the pricing neutrality established above: since the real allocation—and thus the firm's state-contingent cash flows—is identical under both dollar-pricing and peso-pricing in a flexible-price world, the underlying exchange rate exposure remains unchanged. Because the firm faces the same fundamental risk profile regardless of the nominal unit of account, its optimal hedging strategy in the state-contingent case is independent of the invoicing currency.

**Approximation Results** The following result establishes a formal link between preset plans and state-contingent plans, providing the analytical tractability required for our study of currency choice.

**Lemma 2** (Approximation of Preset Allocations)**.** Assume the Hessian of the value function $\widetilde{\Pi}(s_{t+1}) \equiv \Pi_i(\widetilde{p}^{\iota}(s_{t+1}), \widetilde{H}^{\iota}(s_{t+1}); s_{t+1})$ is full rank. Then, up to a second-order approximation of the value function, the preset allocation equals the expected desired allocation. Specifically, the preset prices satisfy:

$$\overline{p}_{i,t}^{\mathrm{USDI}} = \mathbb{E}_t\left[\widetilde{p}^{\mathrm{USDI}}(s_{t+1})\right], \quad \text{and} \quad \overline{p}_{i,t}^{*\mathrm{COPI}} = \mathbb{E}_t\left[\widetilde{p}^{*\mathrm{COPI}}(s_{t+1})\right], \tag{5}$$

and the optimal preset hedging position is identical across invoicing regimes:

$$\overline{H}_{i,t}^{\mathrm{USDI}} = \overline{H}_{i,t}^{\mathrm{COPI}} = \mathbb{E}_t\left[\widetilde{H}(s_{t+1})\right]. \tag{6}$$

Lemma 2 provides an intuitive characterization of the firm's optimal behavior under nominal rigidities. The approximation suggests that preset decisions effectively "target" the mean of their state-contingent counterparts: the firm sets its price equal to the expected desired price in the chosen invoicing currency, and it selects a hedging position equal to the expected desired hedge.

Crucially, while the optimal preset prices are regime-specific due to the different units of account, the optimal hedging position is not. This invariance implies that the preset hedge ratio remains the same under both dollar- and peso-pricing, mirroring the neutrality found in the state-contingent case.

### 2.3 Main Results

To establish the relationship between financial hedging and currency choice in Proposition 1, it is first necessary to define the exchange-rate sensitivity of our state-contingent variables.

**Definition 1** (Exchange-Rate Beta)**.** For any state-contingent variable $\widetilde{x}(s_{t+1})$, we define its exchange-rate beta (conditional on the information set at time $t$) as:

$$\beta_x \equiv \frac{\text{cov}_t(\widetilde{x}(s_{t+1}), s_{t+1})}{\text{var}_t(s_{t+1})}. \tag{7}$$

This exchange-rate beta captures the conditional comovement of a state-contingent variable with the exchange rate. Statistically, $\beta_x$ represents the slope coefficient obtained from a linear projection of the state-contingent variable onto the realized exchange rate.

**Proposition 1** (Sufficient Conditions for Currency Invoicing)**.** Up to a second-order approximation of the value function, a firm chooses dollar pricing if

$$\beta_p + \beta^\tau \ > \ -\frac{1}{2}, \qquad \text{where} \quad \beta_p \equiv \beta_{\widetilde{p}^{\text{USDI}}}, \quad \text{and} \quad \beta^\tau \equiv \frac{\widetilde{\Pi}_{pH}}{\widetilde{\Pi}_{pp}}\,\beta_H. \tag{8}$$

Proposition 1 establishes a sufficient condition for the optimal invoicing decision that depends on two distinct components: the canonical price-stability term, $\beta_p$, and our novel hedging-interaction term, $\beta^\tau$. We unpack the economic intuition behind these terms in the remainder of this section. To ground our analysis, Table 1 provides a high-level comparison between our framework and the benchmark model of Engel (2006).

#### 2.3.1 Intuition for the Canonical Term $\beta_p$

The first component is the classic Engel term, which governs the firm's choice in the absence of hedging. Here, $\beta_p$ represents the desired pass-through into the flexible (state-contingent) price. As established, under flexible pricing, the currency of invoicing is neu-

Table 1: Comparison of Invoicing Frameworks

| **No Hedging (Engel, 2006)** | **Our Hedging Channel** |
|---|---|
| *Panel A: Role of currency invoicing* | |
| An indexing problem. | Beyond the indexing problem. |
| *Panel B: Tracking-error criterion — minimizing tracking error(s) between* | |
| Preset and state-contingent prices. | (a) Preset and state-contingent prices,<br>(b) Preset and state-contingent hedging. |
| *Panel C: pricing and invoicing* | |
| Equivalence result. | Pricing–hedging trade-off. |

tral. The choice becomes meaningful only in the presence of nominal rigidities, as the invoicing regime determines which nominal price remains fixed across exchange-rate states. This identifies the role of currency invoicing as a pure indexing problem, as shown in the left column of Panel A in Table 1.

To understand $\beta_p$, it is helpful to consider two polar cases: if a firm is a pure dollar-pricing type, its desired dollar price is essentially insulated from exchange-rate movements, so $\beta_p = 0$. At the other extreme, if a firm is a pure peso-pricing type, it seeks to keep its peso-denominated price from co-moving with the exchange rate, implying $\beta_{p^*} = 0$. Using the conversion in Lemma 1, this implies that for a peso-pricing firm, the desired price in dollars moves one-for-one with the exchange rate in the opposite direction, so $\beta_p = -1$. Thus, $\beta_p$ serves as an index of where the firm's optimal pricing behavior lies between these two extremes. Values closer to 0 characterize firms that behave like dollar pricers, while values closer to $-1$ characterize those behaving like peso pricers. The threshold of $-1/2$ in Proposition 1 represents the midpoint between these two regimes.

To build further intuition, the firm's objective in Section 2.1 can be recast as a tracking-error criterion (Engel, 2006; Mukhin, 2022). The firm selects the invoicing currency to minimize the expected squared deviation between the preset price and the ex-post desired flexible price. This single-objective optimization, which focuses exclusively on the deviation between preset and state-contingent prices, corresponds to the cell in the left column of Panel B in Table 1. Intuitively, any such deviation reduces firm value relative to the first-best state-contingent plan.

This logic delivers the indexing result of Engel: absent hedging, the invoicing decision is determined by how the desired price co-moves with the exchange rate. If the desired price is relatively more stable when expressed in dollars, the firm prefers dollar-pricing; if it is more stable in peso units, the firm prefers peso-pricing. As summarized in the left column of Panel C in Table 1, this creates a direct equivalence result between the invoicing choice and the desired exchange rate pass-through. Ultimately, the choice of invoicing is a choice of the exchange-rate "loading" of the preset price. The determinants of this stability—such as the sensitivity of marginal costs and markups—are explored further in Section 4.

### 2.3.2 An Irrelevance result: Modigliani and Miller (1958) Meets Engel (2006)

To provide economic context for the new $\beta^{\tau}$ term, we first highlight a useful irrelevance result in Corollary 1. This result allows us to understand our framework as a generalization of two canonical benchmarks.

**Corollary 1** (Irrelevance of Hedging for Invoicing)**.** Up to a second-order approximation of the value function, if $\widetilde{\Pi}_{pH} = 0$, then, holding $\beta_p$ fixed, the availability or cost of financial hedging does not affect the optimal currency choice; equivalently, $\beta^{\tau} = 0$.

From the perspective of the currency invoicing literature, Corollary 1 echoes the core argument in Gopinath and Itskhoki (2022) that financial hedging is irrelevant for currency choice when the hedge is a pure “side bet” that does not alter the firm’s marginal incentives.[8] In our notation, this corresponds to the case where the cross-partial $\widetilde{\Pi}_{pH}$ is zero.

Mathematically, this condition admits a natural interpretation in classic corporate finance as Fisher (1930) separation result: the firm’s production and pricing choices are independent of its hedging and financing decisions. This is the canonical irrelevance benchmark in the spirit of Modigliani and Miller (1958), under which firms can perfectly separate financial risk management from real allocations. In the context of our framework, this separation defines the left column of Table 1: currency invoicing remains a pure indexing problem (Panel A) and the canonical equivalence result between invoicing and price-tracking holds (Panel C).

A key implication of this benchmark is that for hedging to influence currency choice, it must break this MM separation by feeding back into pricing incentives. Whenever hedging is a "side bet" that is irrelevant for the firm’s real value, it is also irrelevant for the invoicing choice. In this sense, the cross-derivative $\widetilde{\Pi}_{pH}$ provides a convenient measure of the extent to which the irrelevance benchmark is violated. When $\widetilde{\Pi}_{pH} \neq 0$, the firm moves to the right column of Table 1, where it must minimize tracking errors for prices and hedging jointly (Panel B).

[8] To quote Gopinath and Itskhoki (2022): “To the extent that the financial hedge is a side bet in the financial market and does not affect the marginal cost of the firm, the currency of price stickiness in the product market is consequential for allocations... In turn, financial frictions do not change the theory of currency choice in the goods market, but may alter the desired price.”

**Existing Theories and Evidence** Taken together, Corollary 1 provides a sharp irrelevance benchmark: hedging affects currency choice only insofar as it alters real allocations on the production side. In our framework, where hedging is an active decision of the firm, this separation—representing the left column of Table 1—is a fragile, knife-edge case rather than the rule. While the benchmark is theoretically illuminating, the independence of pricing and hedging typically breaks down as soon as we move beyond the frictionless ideal.

This motivates a natural question: what economic frictions generate such feedback in practice? A long-standing tradition in corporate finance has been dedicated to studying exactly these departures from the MM independence of financial and real decisions. Theoretically, this separation can be disrupted by managerial motives or tax incentives (Smith and Stulz, 1985), asymmetric information (DeMarzo and Duffie, 1995), financial constraints (Froot et al., 1993; Rampini and Viswanathan, 2010), or contracting frictions (Xie, 2025). Whenever these classic frictions are present, the firm's invoicing decision moves into the right column of Table 1, where the pricing–hedging trade-off becomes a first-order determinant of currency choice.

On the empirical front, a large body of work documents strong two-way interactions between hedging and real decisions (e.g., Rampini et al., 2014). In the specific context of FX hedging, such activities are widespread (Alfaro et al., 2021; Du and Huber, 2024; Hacioglu Hoke et al., 2026) and extend beyond non-financial firms. FX hedging, which serves as a substitute for operational ("real") hedges (Hoberg and Moon, 2017; Gopinath and Stein, 2021), is shaped by firms' foreign asset and liability positions (Liao and Zhang, 2025; Puriya and Bräuning, 2021), and thus comoves with firms' real activities (Adams and Verdelhan, 2022). As a result, fluctuations in the cost of FX hedging have real effects on firms' decisions and value (Jung, 2025; Hommel and Piquard, 2025). With hedging as an active firm decision, this evidence suggests the joint optimization of pricing and hedging better reflects corporate reality.

### 2.3.3 Intuition for the New Term $\beta^\tau$

Crucially, Proposition 1 introduces the additional term $\beta^\tau$, which isolates the incremental role of exchange-rate uncertainty operating through the firm's financial hedging. With endogenous hedging, currency choice can *go beyond* a pure indexing problem because the firm's optimal pricing and financial decisions are fundamentally linked. As summarized in the right column of Panel A in Table 1, this moves the invoicing decision into a regime where currency choice and financial risk management are no longer separable.

One way to interpret $\beta^\tau$ is as an additional tracking-error induced by the hedging decision. In our framework, the firm is no longer concerned exclusively with tracking its desired price; it also cares about tracking its desired hedging position, as any deviation from the optimal risk-management plan reduces firm value. This represents a transition from a single-objective problem to the joint tracking-error criterion shown in the right column of Panel B in Table 1. Because the cross-partial $\widetilde{\Pi}_{pH}$ is nonzero, any deviation in the preset price from its state-contingent target alters the marginal value of the hedge (and vice versa).

The strength of this interaction is captured by the ratio $\widetilde{\Pi}_{pH}/\widetilde{\Pi}_{pp}$, which scales the price adjustment required to offset a marginal change in the profitability of the hedge relative to the curvature of the profit function. This interaction gives rise to the pricing–hedging trade-off listed in the right column of Panel C in Table 1. When these cross-partials are present, the firm may intentionally deviate from the canonical indexing rule to better align its pricing with its financial exposure. Thus, hedging considerations enter the currency-choice condition precisely through $\beta^{\tau}$, breaking the simple equivalence result of the benchmark model.

**Remark 2** (Mapping to the Engel Formula)**.** To bridge our result with the currency-choice literature, we can easily map $\beta^{\tau}$ to the canonical Engel (2006) condition: $\beta = \widetilde{\Pi}_{p\mathbf{x}}^{\top}/\widetilde{\Pi}_{pp} \cdot \beta_{\mathbf{x}}$. In this formulation, $\beta_{\mathbf{x}}$ measures the exchange-rate exposure (co-movement) of the firm's state variables $\mathbf{x}$, while the ratio $\widetilde{\Pi}_{p\mathbf{x}}^{\top}/\widetilde{\Pi}_{pp}$ represents the sensitivity of optimal pricing to those states.

Our framework clarifies how this formula applies when "states" are "endogenous choices". By partitioning $\mathbf{x} = (s, H)^{\top}$, where $s$ is the spot rate and $H$ is the firm's predetermined (but state-contingent) hedge, the formula decomposes as:

$$\beta = \frac{\widetilde{\Pi}_{ps}}{\widetilde{\Pi}_{pp}} \cdot 1 + \frac{\widetilde{\Pi}_{pH}}{\widetilde{\Pi}_{pp}} \cdot \beta_H = \beta_p + \beta^{\tau}.$$

The key conceptual step is treating the firm's optimal hedging plan as an Engel-style state. While $H$ is an optimal plan formed at $t$, its realization at $t+1$ co-moves with the exchange rate according to the firm's state-contingent strategy. One can thus think of $H$ as an auxiliary state variable. This co-movement—captured by $\beta_H$—feeds back into the marginal profitability of pricing. Consequently, $\beta^{\tau}$ is exactly the component of the currency-choice rule that accounts for how the firm's own financial risk-management plan tilts its real pricing incentives.

## 2.4 Discussions

**Remark 3** (Towards a More General Currency-Choice Problem)**.** A brief digression helps highlight a broader point. Our framework focuses on an endogenous hedge position that is chosen at $t$ and is therefore predetermined when the exchange-rate shock realizes. More generally, however, the same logic applies to any endogenous firm choice set prior to exchange-rate realizations—such as quantities (Xie, 2025), contract terms (Antràs, 2003), financing (Antras and Foley, 2015; Bahaj and Reis, 2020), or input sourcing (Arkolakis et al., 2023; Kleinman et al., 2025; Fan and Luo, 2025)—that enters the ex-post pricing problem as a predetermined state.

This perspective clarifies the key sufficient statistics in this broader class of problems. The optimal currency choice is pinned down by two elements as shown in Proposition 1: (i) the local curvature of the profit function, specifically the relevant Hessian evaluated at the desired allocation, which captures the interaction between pricing and other firm decisions; and (ii) the exchange-rate betas of these predetermined variables, which summarize their equilibrium co-movement with the exchange rate.

**Remark 4** (Proof Sketch for Proposition 1)**.** To identify the optimal invoicing regime, we define the value gap $\Delta\Pi^{\text{USDI}-\text{COPI}}$ as the expected profit difference between dollar- and peso-pricing:

$$\Delta\Pi^{\text{USDI}-\text{COPI}} \equiv \mathbb{E}_t\left[\Pi_i(\overline{p}^{\text{USDI}}, \overline{H}^{\text{USDI}}; s_{t+1}) - \Pi_i(\overline{p}^{\text{COPI}}, \overline{H}^{\text{COPI}}; s_{t+1})\right].$$

The firm chooses dollar-pricing if $\Delta\Pi^{\text{USDI}-\text{COPI}} > 0$. We perform a second-order Taylor expansion of $\Delta\Pi^{\text{USDI}-\text{COPI}}$ around the state-contingent (flexible) allocations $(\widetilde{p}, \widetilde{H})$. The first-order terms vanish by the envelope property of the flexible-price problem. The remaining second-order terms decompose into three distinct economic effects:

1. *The Pricing Term (term$_{pp}$):* This captures the standard welfare loss from sticky prices. Following Engel (2006), the tracking error in price across regimes reduces to a covariance between the spot rate and the desired price, which related to $\beta_p$ term.

2. *The Hedging Invariance (term$_{HH}$):* This term involves the difference in the welfare loss from "hedging stickiness" across regimes: $\mathbb{E}_t[(\overline{H}_t^{\text{USDI}} - \widetilde{H}_{t+1})^2 - (\overline{H}_t^{\text{COPI}} - \widetilde{H}_{t+1})^2]$. Crucially, the optimal state-contingent hedging plan $\widetilde{H}$ is determined by the firm's underlying real exposures and the properties of the financial market, which are independent of the nominal unit of account used for pricing. By Lemma 2, the preset hedge $\overline{H}$ in both regimes "targets" this same expected flexible hedge. Consequently,

the hedging-related welfare loss is identical under both dollar- and peso-pricng, and the terms cancel out in the value gap $\Delta\Pi^{\text{USDI}-\text{COPI}}$.

3. *The Interaction Term (term$_{pH}$):* This term captures the feedback between financial and real decisions i.e. $\text{term}_{pH} = -\widetilde{\Pi}_{pH} \cdot \text{cov}_t(\widetilde{H}_{t+1}, s_{t+1})$. The cross-derivative $\widetilde{\Pi}_{pH}$ measures how the marginal profit of pricing shifts with the hedge. One can interpret this as how much financial risks is generated by the choice of preset hedging $\overline{H}$. As we have highlighted before, $\overline{H}$ can be any preset firm choice and the cross-derivative captures its interaction with pricing and hence contributes to the choice of currency.

Collecting these terms and expressing them via exchange-rate betas in Definition 1 and setting $\Delta\Pi^{\text{USDI}-\text{COPI}} > 0$, we recover the modified indexing rule in Proposition 1, which completes the proof.

## 2.5 Taking Stock: From General Theory to Applications

The general framework developed in the previous section provides a unified sufficient condition for optimal currency choice. Proposition 1 suggests that financial hedging shifts the invoicing boundary if it either alters the fundamental exchange-rate pass-through into desired prices ($\beta_p$) or creates a pricing-invoicing tradeoff ($\beta^\tau$) relative to the MM benchmark.

In the remainder of the paper, we operationalize this logic through two structural examples. First, we adopt the classic corporate risk management framework of Froot, Scharfstein and Stein (1993). In this setting, we explicitly compute the pricing-hedging interaction term, $\widetilde{\Pi}_{pH}$, to showcase how financial distress or investment-policy frictions generate an endogenous tilt in pricing incentives.

Second, we consider a model where firms use FX forward contracts to hedge idiosyncratic cash-flow risk, bridging our theory with the international pricing-to-market literature (e.g., Atkeson and Burstein, 2008a). The purpose of this example is twofold: it demonstrates that hedging can endogenously shift the exchange-rate pass-through into the desired price ($\beta_p$), and it yields two distinct empirical predictions that can be testable using firm-level data.

**Exchange Rate Dynamics and Hedging Frictions.** Before proceeding to these examples, we specialize our environment by introducing the exchange-rate dynamics and the mechanics of FX forwards. We assume the exchange rate follows a random walk:

$$s_{t+1} = s_t + \varepsilon_{t+1}, \quad \text{with} \quad \varepsilon_{t+1} \sim \mathcal{N}(0, \sigma_s^2). \tag{9}$$

This specification aligns with the empirical literature on exchange-rate behavior at short horizons and simplifies the interpretation of $\beta$ as a static hedge ratio.

Central to our application is the use of FX forward contracts. A firm with a foreign-currency cash inflow hedges its exposure by selling that currency forward for delivery at $t+1$.

**Remark 5** (Hedging with FX Forwards)**.** To fix ideas, consider a firm receiving dollar revenues at date $t+1$. The firm is effectively *long* dollars: a dollar depreciation (a fall in $S_{t+1}$) reduces the domestic value of its cash flows. The firm hedges this risk by *selling dollars forward* at date $t$, committing to deliver dollars at $t+1$ in exchange for pesos at the predetermined rate $F_t$. If the dollar depreciates ($S_{t+1} < F_t$), the firm is protected because it converts the hedged portion of its receipts at the higher, pre-agreed forward rate rather than the unfavorable realized spot rate.

In a frictionless market, the forward price $f_t$ would equal the expected future spot rate $\mathbb{E}_t[s_{t+1}]$. However, we introduce a hedging friction $\tau > 0$, representing a "haircut" or transaction cost:

$$f_t = \mathbb{E}_t[s_{t+1}] - \tau. \tag{10}$$

where, without loss of generality, we shut down the interest differentials across two countries $i_t - i_t^* = 0$. The parameter $\tau$ serves as a reduced-form representation of institutional frictions, including bid–ask spreads (Moskowitz et al., 2024), OTC search frictions (Babus and Kondor, 2018), margin costs (Bolton et al., 2013; Hommel and Piquard, 2025), or intermediary balance-sheet constraints (Gabaix and Maggiori, 2015; Du et al., 2018; Barbiero et al., 2024; Dao et al., 2025). By varying $\tau$, we trace how the cost of financial risk management propagates into real pricing decisions.

## 3 Example I: Froot, Scharfstein and Stein (1993)

To illustrate the "structural roadmap" connecting corporate finance to currency choice, we specialize our framework using the classic model of risk management by Froot et al. (1993). In this setting, the violation of MM irrelevance stems from a convex cost of external finance, which forces the exporter to coordinate its pricing and hedging decisions to protect its internal liquidity.

### 3.1 Set Up

**Environment** At time $t$, the firm is endowed with a dollar-denominated asset $K_t = 1$ and must commit to three preset decisions: its price $P_t$, its hedging ratio $H_t$, and its invoicing currency $\iota$. The state of the world is revealed at $t+1$ with the realization of the spot exchange rate $S_{t+1}$. Consequently, the peso-denominated value of the firm's internal funds, $K_{t+1}$, is determined by the portion of the asset exposed to the spot market versus the portion converted at the preset forward rate $F_t$:

$$K_{t+1} = (1 - H_t)S_{t+1} + H_t F_t \tag{11}$$

This specification captures the firm's financial flexibility: an unhedged exporter ($H_t = 0$) leaves its internal liquidity entirely subject to exchange rate volatility, whereas a fully hedged firm ($H_t = 1$) can effectively lock in its internal capital at the forward rate $F_t$ known at time $t$. [9]

To capture the structural tension between investment requirements and internal liquidity, we follow Froot et al. (1993) by assuming that the demand shifter $D^0_{t+1}$ is a stochastic variable correlated with the exchange rate $S_{t+1}$. Specifically, let $D^0_{t+1} = \overline{D}^0 + \alpha(S_{t+1} - S_t)$, where the parameter $\alpha$ governs the covariance between demand shocks and currency fluctuations.

The economic intuition hinges on the sign of $\alpha$. When $\alpha < 0$, the firm faces a pro-cyclical financing squeeze: a "weak" dollar (low $S_{t+1}$) coincides with a positive demand shock. In the absence of hedging, internal liquidity is most scarce precisely when the firm's need to expand production—and thus its need for investment capital—is at its peak. Consistent with the logic of Froot et al. (1993), this state-contingent mismatch generates a powerful incentive for the firm to hedge. By utilizing the forward market, the firm stabilizes its internal funds to ensure they are sufficient to fund investment during high-demand states, effectively decoupling its real production capacity from nominal exchange rate volatility.

**Firm's Objective** For a fixed invoicing regime $\iota$, the firm maximizes expected terminal value, defined as operating revenues minus the quadratic costs of equity issuance ($E_{t+1}$) parameterized by $\kappa$: $\mathbb{E}_t[P_t Y_{t+1} - \frac{\kappa}{2} E^2_{t+1}]$. This objective implies that the firm is effectively

[9] In Equation (11), we implicitly restrict cash flows from the hedging contract to be used only as internal funds, ruling out arbitrage through these contracts. This restriction is equivalent to a three-period environment: the firm first enters the hedging contract; the exchange rate is then realized and internal funds are determined; and production takes place only in the final stage. Crucially, we assume that the firm values only terminal cash flows and has no storage technology between the intermediate and final stages (Lorenzoni, 2008). Consequently, even the flexible firm would never use hedging contracts to obtain resources in excess of the flexible benchmark $\widetilde{K}_{t+1}$.

risk averse with respect to equity issuance, and therefore to the availability of internal funds. In particular, the firm seeks to align internal liquidity with investment needs so as to reduce its reliance on costly external funds $E_{t+1}$.

We characterize the firm's environment with a downward-sloping demand curve:

$$Y_{t+1} = 2D^0_{t+1} - D^1 P_t, \tag{12}$$

where $D^0_{t+1}$ is the stochastic demand shifter and $D^1$ represents the price sensitivity of demand, which is stable over time. The firm utilizes a production technology where output equals investment, $Y_{t+1} = I_{t+1}$, and a funding constraint where all investment must be financed via internal funds $K_{t+1}$ or external equity $E_{t+1}$, such that $I_{t+1} = K_{t+1} + E_{t+1}$.

Substituting these constraints and the law of motion for $K_{t+1}$ from Equation (11), the firm's problem reduces to:

$$\max_{P_t, H_t, \iota} \mathbb{E}_t \left[ P_t \underbrace{(2D^0_{t+1} - D^1 P_t)}_{Y_{t+1}} - \frac{\kappa}{2} \left( \underbrace{2D^0_{t+1} - D^1 P_t}_{Y_{t+1}} - \underbrace{[(1 - H_t) S_{t+1} + H_t F_t]}_{K_{t+1}} \right)^2 \right] \tag{13}$$

In the presence of financial frictions ($\kappa > 0$), pricing and hedging decisions are no longer separable. The optimal price depends on the expected financing gap, while the optimal hedge is chosen to minimize the variance of that gap, particularly when demand shocks are correlated with the exchange rate.

## 3.2 Solution

**First-Order Conditions** To characterize the interaction between financial frictions and real decisions, we derive the first-order conditions under the state-contingent benchmark. The firm faces a fundamental trade-off: it must balance standard marginal revenue from pricing against the cost of financing any potential shortfall between output and internal funds. This mismatch represents the *funding gap*—the extent to which production $Y_{t+1}$ exceeds available liquidity generated by the internal funds $K_{t+1}$.

The first-order conditions (FOCs) make this structural dependence transparent:

$$[\widetilde{P}] \quad \widetilde{\Pi}_P = 0 \implies \underbrace{(2D^0_{t+1} - 2D^1 \widetilde{P}_{t+1})}_{\text{marginal revenue}} - \kappa \underbrace{[2D^0_{t+1} - D^1 \widetilde{P}_{t+1} - \widetilde{K}_{t+1}]}_{\text{funding gap}} (-D^1) = 0 \tag{14}$$

$$[\widetilde{H}] \quad \widetilde{\Pi}_H = 0 \implies -\kappa \underbrace{[2D^0_{t+1} - D^1 \widetilde{P}_{t+1} - \widetilde{K}_{t+1}]}_{\text{funding gap}} (S_{t+1} - F_t) = 0 \tag{15}$$

The pricing condition (14) dictates that the marginal benefit of raising prices (the marginal revenue term) must be weighed against its effect on the financing gap. Because changing the price alters demand, it directly dictates the volume of output that requires financing. The condition for hedging (15) is also straightforward: it implies that the firm chooses its hedging ratio so that available internal funds $\widetilde{K}_{t+1}(S_{t+1})$ align with desired output $\widetilde{Y}_{t+1}(S_{t+1})$ state by state. In this sense, hedging serves as the primary instrument to close the financing gap.

Solving this system yields a clean set of optimal decisions. The optimal hedge satisfies the state-contingent condition, closing the funding gap entirely:

$$(F_t - S_{t+1})\widetilde{H}_{t+1}(S_{t+1}) = D^0_{t+1} - S_{t+1} \tag{16}$$

The left-hand side represents the total transfer from the forward contract in a given state: for each unit of the desired forward position $\widetilde{H}_{t+1}$, the contract delivers a net payoff of $(F_t - S_{t+1})$, where $F_t$ is the locked-in payoff at $t$ and $S_{t+1}$ represents the opportunity cost of the spot market. The desired price is pinned down by setting marginal revenue to zero, $\widetilde{P}(S_{t+1}) = D^0_{t+1}/D^1$ given the funding gap is always closed by the hedging position.

This condition ensures that internal funds fluctuate with the exchange rate in precisely the manner required to support the desired level of output. Intuitively, the firm seeks a larger cash windfall when demand ($D^0_{t+1}$) is high to facilitate the expansion of production. Conversely, when the dollar appreciates (higher $S_{t+1}$), the firm's existing dollar endowment revalues upward; the optimal hedge then accounts for this natural increase in wealth, ensuring that the total liquidity available to the firm remains aligned with its investment needs across all states.

**Intuition of $\widetilde{\Pi}_{pH}$** The core of our departure from the MM benchmark is captured by the non-separability of the firm's choice variables. To see this formally, we take the derivative of the pricing first-order condition $\widetilde{\Pi}_p$ in Equation (14) with respect to the hedging ratio $H$ and pricing $P$, respectively, which yields the cross-partial derivative:

$$\widetilde{\Pi}_{PH}(S_t) = -D^1\kappa(F_t - \mathbb{E}_t[S_{t+1}]) = D^1\kappa\tau, \quad \widetilde{\Pi}_{PP}(S_t) = -\left[2D^1 + \kappa(D^1)^2\right] \tag{17}$$

where the second equality in $\widetilde{\Pi}_{PH}(S_t)$ comes from the assumption about the forward haircut in Equation (10) [10] This term represents the fundamental interaction in our model.

[10] In Proposition 1, the relevant cross-derivative is taken with respect to the log price $p = \log P$ rather than the level price $P$. By the chain rule, $\widetilde{\Pi}_{pH}(S_t) = \widetilde{P}\,\widetilde{\Pi}_{PH}(S_t)$ and $\widetilde{\Pi}_{pp}(S_t) = \widetilde{P}\,\widetilde{\Pi}_P(S_t) + \widetilde{P}^2\,\widetilde{\Pi}_{PP}(S_t) = \widetilde{P}^2\,\widetilde{\Pi}_{PP}(S_t)$, where the first term in the second expression vanishes by the first-order condition.

In the MM limit where external finance is frictionless ($\kappa = 0$), the cross-partial vanishes, and the firm's production decisions become independent of its financial risk management. However, when $\kappa > 0$, the hedging ratio $H$ directly shifts the marginal cost of output by altering the expected burden of the financing gap. This structural link ensures that any change in the cost of hedging ($\tau$) or the cost of external financing ($\kappa$) will endogenously feed back into its pricing and invoicing strategy.

The magnitude of this non-separability is strictly proportional to both the cost of external finance $\kappa$ and the forward haircut $\tau$. Under the MM benchmark ($\kappa = 0$), the cross-partial vanishes, and production decisions become independent of financial risk management. Here, however, that separation breaks down: because the hedging ratio $H$ determines the level of available internal funds, it directly shifts the marginal cost of output, thereby feeding back into the optimal pricing decision. This structural link is what activates the $\beta_\tau$ channel in the firm's ultimate currency-invoicing choice.

## 4 Example II: Hedging Cash Flow

In this section, we embed an explicit FX forward contract into a standard international pricing-to-market model as in Amiti et al. (2014), where firms must manage the currency risk arising from their operational cash flows from both revenues and costs. This framework allows us to characterize two primary channels through which financial risk management interacts with the real economy.

First, beyond the fundamental pricing-hedging tradeoff characterized by $\beta^\tau$, we show that FX hedging endogenously alters the exchange rate pass-through into prices, captured by $\beta_p$; as a firm manages a greater portion of its exchange rate risk through financial markets, its optimal pricing sensitivity to spot rate fluctuations shifts accordingly. Second, this environment yields closed-form solutions for the key theoretical objects $\beta_p$ and $\beta^\tau$ in Proposition 1. These solutions provide sharp, firm-level predictions regarding the relationship between currency exposures, hedging costs, and invoicing choices, which can be mapped to the data.

### 4.1 Set up

**Environment** At time $t$, a Colombian firm determines its export price $P_t$, its hedging ratio $H_t$, and its invoicing currency $\iota$. Upon observing the exchange rate at $t+1$, the firm implements its production plans, incurs costs, and ships the goods to the destination. US consumers purchase the goods, generating revenue in dollars. The firm then converts its net dollar cash inflow—defined as the difference between dollar-denominated revenues and foreign input costs—back into pesos at the effective conversion rate determined by its hedging position.

**Production and Demand** The firm produces variety $i$ with productivity $A_i$ using a Cobb-Douglas technology with domestic labor $L_i$ and foreign intermediate inputs $M_i$:

$$Y_i = A_i L_i^{1-\alpha_i} M_i^{\alpha_i} \tag{18}$$

where $\alpha_i \in [0,1]$ denotes the foreign input share. [11]

On the demand side, the firm faces a destination-market demand schedule derived from Constant Elasticity of Substitution (CES) preferences. [12] Normalizing the destination price index and aggregate expenditure to unity, variety-level demand is given by:

$$Y_{i,t+1} = \left(P_{i,t+1}^{\text{dock}}\right)^{-\eta} \tag{19}$$

where $\eta > 1$ represents the constant price elasticity of demand. A higher $\eta$ reflects lower markup, measured by $\eta/(\eta-1)$.

**Firm's Objectives** The firm chooses its preset price and hedging positions to maximize its expected value:

$$\max_{P^\iota, 0 \le H_t \le 1} \mathbb{E}_t \left[\Pi\left(X_{i,t+1}^\iota(S_{t+1})\right)\right] \tag{20}$$

where $X_{i,t+1}^\iota$ denotes the total cash flow in pesos under invoicing regime $\iota$. The peso-denominated cash flow is defined as:

$$X_{i,t+1}^\iota(S_{t+1}) = \underbrace{\left[(1-H_{i,t})S_{t+1} + H_{i,t}F_t\right]}_{\text{Effective Conversion Rate}} \times \underbrace{\left(P_{i,t}^\iota Y_{i,t+1} - \mathfrak{P}^M M_{i,t+1}\right)}_{\text{Net USD Cash Inflow}} - \underbrace{W^* L_{i,t+1}}_{\text{Domestic Labor Cost}} \tag{21}$$

First, the firm pays its domestic labor cost $W^* L_{i,t+1}$ directly in pesos, creating a liability naturally denominated in the local currency. Second, the firm's operational margin—the surplus of dollar revenues $P_{i,t}^\iota Y_{i,t+1}$ over dollar-priced inputs $\mathfrak{P}^M M_{i,t+1}$—is exposed to exchange rate risk. By selecting $H_{i,t}$, the firm determines the extent to which this net dollar

[11] We assume constant returns to scale to ensure that the marginal cost remains independent of the scale of production, thereby ruling out macroeconomic complementarities arising from feedback between a firm's pricing and its own marginal cost (Galí, 2015).

[12] Our framework can be extended in a straightforward way to an international pricing-to-market model, as in Atkeson and Burstein (2008a); Amiti et al. (2014), by incorporating Kimball (1995) demand. For simplicity, we work with CES demand.

surplus is insulated via the forward rate $F_t$ versus exposed to the spot rate $S_{t+1}$. Given the concavity of $\Pi(\cdot)$, the firm is endogenously risk-averse, providing a structural incentive to stabilize its peso-denominated margins through both pricing and forward market positions.

Crucially, the effective conversion rate, $(1 - H_{i,t})S_{t+1} + H_{i,t}F_t$ creates a wedge on the USD cash inflow when converting USD back to peso, breaking the MM separation theorem. In a no-hedging benchmark, the firm's pricing decision is determined solely by the destination-market markup and the marginal cost expressed in a common currency. However, in this setting, the conversion wedge enters the pricing first-order condition directly as a scaling factor on the marginal revenue of every dollar earned abroad. Consequently, the firm's optimal price $P_{i,t}$ is no longer independent of its financial hedging strategy $H_{i,t}$; the real pricing-to-market decision and the financial risk-management decision are tied together.

## 4.2 Solution

**First-Order Condition** The FOC for price is given by

$$\underbrace{\left[(1 - H_{i,t})S_{t+1} + H_{i,t}F_t\right]^{1-\alpha_i}}_{\text{effective conversion wedge}} P_i = \underbrace{\frac{\eta}{\eta - 1}}_{\text{markup}} \times \underbrace{\frac{1}{A_i}\left(\frac{\mathfrak{P}^M}{\alpha_i}\right)^{\alpha_i}\left(\frac{W^*}{1-\alpha_i}\right)^{1-\alpha_i}}_{\text{marginal cost}} \tag{22}$$

In a canonical closed-economy model, the price is simply a constant markup over the marginal cost. Under the Cobb-Douglas technology specified in Equation (18), the marginal cost is strictly decreasing in productivity $A_i$, as higher efficiency lowers the units of inputs required per unit of output. Conversely, the marginal cost increases in the costs of the input bundle: specifically, it rises with both the domestic wage $W^*$ and the foreign intermediate price $\mathfrak{P}^M$, with the sensitivities determined by the respective labor share $(1 - \alpha_i)$ and foreign input share $(\alpha_i)$.

In a benchmark case without financial hedging ($H_{i,t} = 0$), the conversion wedge simplifies to the spot exchange rate $S_{t+1}^{1-\alpha_i}$, reflecting the simple translation of foreign dollar revenues into the local peso currency. The exponent $1 - \alpha_i$ in the conversion wedge captures the net exchange rate exposure of the firm's operating margin. Specifically, the power of 1 represents the full exposure of dollar-denominated export revenues, which must be translated into pesos to meet domestic obligations. However, for firms utilizing foreign intermediate inputs, a portion of this exposure is offset by a "natural hedge" on the cost side. Because the price of these inputs $\mathfrak{P}^M$ is also denominated in dollars, a dollar appreciation (high $S_{t+1}$) simultaneously also increases cost of production converted into peso.

The resulting effective exposure is thus reduced to the share of domestic value-added, $1 - \alpha_i$. Consequently, firms with a higher foreign input share $\alpha_i$ possess a stronger natural buffer, making their internal peso-margins inherently less sensitive to exchange rate volatility even in the absence of financial derivatives.

However, the introduction of the *effective conversion wedge*—defined as an endogenous weighted average of the spot and forward rates—represents the critical departure from the canonical model. This structure implies that the firm's pricing decision $P_i$ is no longer determined solely by real production constraints and destination-market demand; instead, it is directly scaled by the firm's financial hedging position $H_{i,t}$. This non-separability formally breaks the MM separation theorem, as the firm's real pricing-to-market strategy becomes fundamentally intertwined with its financial risk-management decisions.

**Exchange-Rate Pass-through into Prices** A direct implication of the pricing condition in Equation (22) is that the inclusion of the conversion wedge fundamentally alters the sensitivity of the optimal price $P_i$ to exchange rate fluctuations $S_{t+1}$ compared to the non-hedging benchmark in Amiti et al. (2022). In the standard framework, the pass-through is determined by the cost-side exposure $\alpha_i$. In our setting, however, the financial hedging position introduces an additional state-contingent elasticity. To see this more clearly, taking logs and differentiating the first-order condition (22) with respect to $s_{t+1}$ yields the following characterization of the desired export price at the US dock in dollar.

**Lemma 3** (ERPT into Desired Price)**.** Desired price in the destination market denominated in dollar follows

$$d\widetilde{p}_{i,t+1} = -(1 - \mathring{\alpha}_i - \mathring{\tau}_i) ds_{t+1}, \quad \text{where} \quad \mathring{\alpha}_i = \alpha_i, \quad \mathring{\tau}_i = (1 - \alpha_i)\mathbf{1}(s_{t+1} < f_t). \tag{23}$$

The first term in the bracket, 1, indicates the benchmark of complete pass-through. Intuitively, for a firm that sets prices in its producer currency (PCP), uses entirely domestic inputs, and without access to hedging ($\mathring{\alpha}_i = 0, \mathring{\tau}_i = 0$), the desired destination price moves one-for-one with the exchange rate. In this case, the firm maintains a constant price in peso, allowing the dollar price to absorb the full volatility of the exchange rate shock.

In contrast to the full ERPT benchmark, two distinct channels lead to incomplete pass-through in our framework. First, the *marginal cost channel* ($\mathring{\alpha}_i$) reflects the fact that a depreciation of the peso increases the cost of imported intermediates. Similar to Amiti et al. (2022), we can proxy for this using the foreign input share in total variable costs. Firms with a high $\alpha_i$ exhibit naturally lower pass-through into peso prices because their marginal costs are already partially denominated in the same currency as their revenues.

Second, a unique feature of our setting is the *hedging channel*, captured by $\mathring{\tau}_i$. This term characterizes how the breakdown of the MM separation theorem fundamentally alters real pricing behavior. When a firm is hedged, the "effective" exchange rate used to convert marginal revenue becomes a state-contingent weighted average of the spot and forward rates. In states where the domestic currency is strong ($s_{t+1} < f_t$), the forward contract provides a windfall that effectively lowers the firm's shadow marginal cost in peso terms. This financial shift forces the firm to adjust its desired price beyond what is dictated by physical input costs alone.

In this way, our framework provides a formal departure from the benchmark described in Gopinath and Itskhoki (2022), where financial hedging is typically viewed as a "side bet" that leaves the firm's desired price and marginal cost unaffected. In our model, the introduction of the *effective conversion wedge* implies that the firm's pricing first-order condition is no longer separable from its hedging position. Consequently, the intensity of exchange rate pass-through is not merely a function of production technology, but is endogenously shaped by the firm's financial risk management strategy.

**Remark 6** (Empirical Test of Exchange-Rate Pass-through)**.** The result in Lemma 3 delivers our first empirical test. However, because the desired price $\widetilde{p}$ is not directly observable to the econometrician, we add more structures on the nominal rigidities to map theoretical pass-through into realized price dynamics.

Following the standard international macro literature (Gopinath et al., 2010; Amiti et al., 2019), we assume Calvo price adjustment: in each period, a firm receives an opportunity to reset its price with probability $1 - \nu$, while with probability $\nu$, the price remains fixed at its preset level.Let $\mathrm{d}\mathring{p}_i^\iota$ denote the change in firm $i$'s *realized* dock price denominated in dollars, the destination currency, we can decompose the the change in realized price into the following components

$$\mathbb{E}[\mathrm{d}\mathring{p}_i^\iota] = -\left[ \underbrace{\mathrm{d}s_t}_{\text{(i) full PT}} + \underbrace{\nu\,\mathbf{1}(\iota = \text{USDI})\,\mathrm{d}s_t}_{\text{(ii) no price adj.}} + \underbrace{(1-\nu)(\mathring{\alpha}_i + \overline{\mathring{\tau}}_i)\mathrm{d}s_t}_{\text{(iii) flexible price adj.}} \right], \tag{24}$$

where $\overline{\mathring{\tau}}_i = \mathbb{E}_t[\mathring{\tau}_i] = (1-\alpha_i)\Phi\left(-\frac{\tau}{\sigma_s}\right)$ and $\Phi(\cdot)$ and $\phi(\cdot)$ representing the CDF and PDF of standard normal distribution.

Equation (24) decomposes observed incomplete ERPT into three distinct economic channels. The first term, $\mathrm{d}s_t$, establishes the complete pass-through benchmark: a firm pricing in peso (PCP) with purely domestic inputs and no financial frictions would adjust its destination price one-for-one with the exchange rate.

The second term, premultiplied by the stickiness parameter $\nu$, isolates the *direct effect* *of nominal rigidities*. This effect occurs conditional on no price adjustment and results in zero pass-through of the exchange rate under dollar pricing. As in Amiti et al. (2022), the greater the extent of price stickiness $\nu$, the more the realized ERPT is dominated by the firm's initial invoicing choice.

The third term, premultiplied by $(1-\nu)$, isolates the *flexible-price determinants* of ERPT, conditional on a price adjustment occurring. As emphasized by Lemma 3 , this reflects the structural exposure of the firm's marginal cost to foreign exchange ($\mathring{\alpha}_i$) and, crucially, the additional damping effect induced by the firm's financial hedging strategy ($\overline{\mathring{\tau}}_i$).

In the short run (high $\nu$), we expect the realized ERPT to be driven primarily by the currency of invoicing. However, as we consider longer time horizons where prices become more flexible ($\nu \to 0$), the relative weight in Equation (24) shifts away from the sticky-price term and toward the real and financial determinants, $\mathring{\alpha}_i$ and $\overline{\mathring{\tau}}_i$, which can be directly tested Jordà (2005) local projection by different horizons.

**Currency Choice** In our framework, financial hedging strategies fundamentally shift the incentives for currency choice by altering both $\beta_p$, and, $\beta^\tau$. In this example, we obtain the following closed-form expressions for these determinants.

**Proposition 2.** Up to a second-order approximation of the value function, firm $i$ chooses dollar pricing if $\beta_p + \beta^\tau > -\frac{1}{2}$, up to a second-order approximation of the value function, where:

$$\beta_p = -(1-\alpha_i - \overline{\mathring{\tau}}_i), \quad \beta^\tau = (1-\alpha_i)\frac{\tau}{\sigma_s}\phi\left(\frac{\tau}{\sigma_s}\right)$$

and the expected hedging wedge is $\overline{\mathring{\tau}}_i = \mathbb{E}_t[\mathring{\tau}_i] = (1-\alpha_i)\Phi\left(-\frac{\tau}{\sigma_s}\right)$, with $\Phi(\cdot)$ and $\phi(\cdot)$ representing the CDF and PDF of standard normal and $S_t$ normalized to 1.

The closed-form solutions for $\beta_p$ and $\beta^\tau$ characterize two distinct channels through which financial hedging affects the firm's optimal currency choice. The first term, $\beta_p$, represents the *exchange-rate pass-through channel*. It captures how the inclusion of the conversion wedge alters the sensitivity of the firm's desired price to exchange rate fluctuations, which affects the currency choice (Gopinath and Itskhoki, 2022). The second term, $\beta^\tau$, represents the *pricing-invoicing tradeoff* highlighted in Corollary 1.

The following result characterizes the conditions under which pricing-hedging tradeoff becomes neutralized, echoing our Corollary 1 in Section 2. By studying this example, we effectively recover the classic benchmarks of the international macro literature.

**Corollary 2.** Up to a second-order approximation of the value function, holding $\beta_p$ fixed, financial hedging does not affect currency choice if either hedging is costless $\tau = 0$ or hedging is infeasible $\tau \to \infty$.

The intuition behind Corollary 2 lies in the two extremes where financial frictions cease to influence the firm's strategic trade-off between two types of risk managements: invoicing and FX hedging. In both cases, $\beta^\tau$, captures the tracking-error induced by endogenous hedging decision, vanishes, and we recover the classic neutrality benchmarks.

First, when hedging is infeasible ($\tau \to \infty$), the cost of entering a forward contract is prohibitively high. In this regime, the firm's financial margin is effectively closed, leaving the pricing and invoicing decisions as the only available tools for managing currency risk. Consequently, the model converges to the standard framework in Engel (2006), where the currency choice is driven solely by the volatility of the real operating margin.

Second, when hedging is costless ($\tau = 0$), the firm can perfectly and frictionlessly insulate its operating margins using financial derivatives. Because the marginal cost of adjusting the hedge position is zero, the financial "fix" no longer creates a distortion in the firm's value function or its real pricing decisions. This leads to a form of MM indifference: if currency risk can be mitigated at zero cost through the capital market, the specific nominal denomination of the price tag becomes irrelevant to the firm's total value at the margin. This result echoes the irrelevance theorem discussed in Xie (2025); in that framework, even when MM is violated due to underlying contracting frictions, hedging remains orthogonal to the invoicing decision as long as financial markets are complete and hedging is frictionless.

The novel contribution of our framework lies in the intermediate range. For any finite, non-zero hedging cost, the firm faces a fundamental trade-off where financial risk management is beneficial but costly. This friction generates a strictly positive $\beta^\tau$, which acts as an extensive-margin shift that encourages firms to adopt dollar pricing relative to the standard frictionless benchmark. Having established this extensive-margin effect in Corollary 2, a natural question arises: how do marginal changes in the cost of financial hedging ($\tau$) further shift the firm's optimal currency choice? We characterize these comparative statics in the following proposition.

Having characterized the extensive margin of the hedging decision in Corollary 2, a natural question arises: how do changes in the cost of financial hedging ($\tau$) shift the firm's optimal currency choice? We summarize our finding in the Corollary 3.

**Corollary 3.** Suppose $\tau > \sigma_s$. Holding all other parameters constant, a decline in the dollar hedging cost $\tau$ increases the likelihood that a firm adopts dollar pricing. Furthermore, this effect is heterogeneous: the shift toward dollar pricing is strictly stronger for firms with higher dollar exposure (i.e., those with a high $1 - \alpha_i$).

For a firm adopting dollar pricing, the primary source of risk is a depreciation of the dollar, which reduces the domestic purchasing power of its foreign revenues. A lower hedging cost $\tau$ allows the firm to more efficiently mitigate this downside risk using forward contracts, thereby increasing the relative appeal of dollar-denominated invoicing.

This result stems from the fact that the hedging cost $\tau$ and the natural hedge $\alpha_i$ act as *substitutes* in stabilizing the firm's home-currency margins. When $\tau$ falls, the "financial barrier" to stabilizing dollar revenues is lowered, providing a disproportionate benefit to firms that lack a natural hedge ($\alpha_i = 0$). Because these firms rely more heavily on the financial market to manage exchange rate risk, the reduction in transaction costs provides a stronger incentive for them to switch toward dollar pricing.

**Remark 7** (Empirical Test of Currency Choice)**.** The comparative statics in Corollary 3 provide a structural basis for a firm-level predictive regression of currency invoicing, following the empirical tradition of Amiti et al. (2022). Specifically, our model predicts that the probability of a firm adopting dollar invoicing is a joint function of its financial hedging efficiency and its production characteristics. In the data, the hedging cost $\tau$ can be identified using transaction-level FX forward data such as Alfaro et al. (2021); Hacioglu Hoke et al. (2026); Hommel and Piquard (2025); Fraschini and Terracciano (2023), while the firm's foreign input share $\alpha_i$ is obtained from product-level customs records as in Gopinath et al. (2020); Amiti et al. (2014). To achieve identification, a reduction in the hedging cost $\tau$ can be mapped to structural reforms or market shifts that exogenously alter the supply and demand for FX forward contracts, such as the episode in Jung (2025), Keller (2024), and Alfaro et al. (2021). This mapping allows us to understand the interaction between financial hedging and natural hedging determines the global currency of trade.

# 5 Concluding Remarks

This paper shows that currency invoicing and FX hedging are two sides of the same risk-management problem. In the presence of imperfect hedges and deviations from Modigliani and Miller (1958) irrelevance between pricing and hedging decisions, firms do not choose their invoicing currency solely to stabilize preset prices; they also choose it to manage the interaction between nominal rigidities and financial exposures. This creates a new pricing–hedging trade-off and implies that hedging shapes currency choice both directly and indirectly, by shifting exchange-rate pass-through into desired prices. Moreover, the sensitivity of currency switching to hedging costs is heterogeneous across firms: it is greater for firms with stronger departures from MM, that is, for firms with a tighter connection between production and hedging decisions.

More broadly, our results suggest that dollar dominance in finance and in trade is mutually reinforcing: deep dollar hedging markets make dollar pricing more attractive. By linking corporate risk management to the currency composition of international trade, our framework provides a microfoundation for how financial-market structure can shape the international monetary system. This linkage also delivers two policy implications. For highly dollarized emerging markets, policymakers often seek to deepen FX markets in order to improve firms' ability to manage dollar risk. Yet doing so may unintentionally encourage greater dollar invoicing, thereby undermining efforts to promote the domestic currency internationally. For the United States, our results suggest that its central role in dollar intermediation may help sustain the dollar's dominant international status, beyond the lender-of-last-resort role emphasized in work on swap lines such as Bahaj and Reis (2020).

Some caveats are in order. From the theoretical perspective, there are two natural avenues for future work. First, rooted on Proposition 1, we provide microfoundations for two examples from corporate risk management to illustrate how hedging and invoicing decisions are intertwined in Section 3 and Section 4. Additional examples will enrich the scope of the framework and help us understand whether different hedging motives generate different predictions for currency invoicing. Second, an alternative approach is to remain agnostic about the precise underlying hedging motives and instead characterize Proposition 1 using a small set of estimable sufficient statistics, in the spirit of Kurtzman and Zeke (2023), who use a sufficient-statistics approach to study the aggregate implications of deviations from MM irrelevance. On the empirical side, we plan to leverage a unique and proprietary firm-level dataset, exploit quasi-random variation in hedging costs driven by unexpected reforms, and estimate the key objects needed to evaluate our theoretical predictions. These directions are left to our ongoing and follow-up work.

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# A Proof, Derivations, and Extensions

## A.1 Proof of Lemma 2

*Proof.* Fix a currency regime $\iota \in \{\text{USDI}, \text{COPI}\}$. Preset choices $(\overline{p}_t^\iota, \overline{H}_t^\iota)$ satisfy the following FOCs:

$$\mathbb{E}_t\Big[\Pi_p^\iota(\overline{p}_t^\iota, \overline{H}_t^\iota; s_{t+1})\Big] = 0, \tag{A.1}$$

$$\mathbb{E}_t\Big[\Pi_H^\iota(\overline{p}_t^\iota, \overline{H}_t^\iota; s_{t+1})\Big] = 0. \tag{A.2}$$

Let $\widetilde{x}_{t+1}^\iota \equiv (\widetilde{p}_{t+1}^\iota, \widetilde{H}_{t+1}^\iota)$ denote the state-by-state desired allocation at $s_{t+1}$, and define $\widetilde{\Pi}^\iota(s) \equiv \Pi^\iota(\widetilde{x}^\iota(s) \mid s)$.

Now a first-order Taylor expansion of $\Pi_p^\iota(\cdot \mid s_{t+1})$ around $\widetilde{x}_{t+1}^\iota$ gives

$$\Pi_p^\iota(\overline{x}_t^\iota \mid s_{t+1}) = \underbrace{\Pi_p^\iota(\widetilde{x}_{t+1}^\iota \mid s_{t+1})}_{=0} + \widetilde{\Pi}_{pp}^\iota(s_{t+1})\big(\overline{p}_t^\iota - \widetilde{p}_{t+1}^\iota\big) + \widetilde{\Pi}_{pH}^\iota(s_{t+1})\big(\overline{H}_t^\iota - \widetilde{H}_{t+1}^\iota\big)$$
$$+ \mathcal{O}\big(\|\overline{x}_t^\iota - \widetilde{x}_{t+1}^\iota\|^2\big). \tag{A.3}$$

To replace derivatives evaluated at $s_{t+1}$ by those at $s_t$, consider for example

$$\widetilde{\Pi}_{pH}^\iota(s_{t+1})\big(\overline{H}_t^\iota - \widetilde{H}_{t+1}^\iota\big) = \widetilde{\Pi}_{pH}^\iota(s_t)\big(\overline{H}_t^\iota - \widetilde{H}_{t+1}^\iota\big) + \mathcal{O}(\|s_{t+1} - s_t\|\,\|\overline{x}_t^\iota - \widetilde{x}_{t+1}^\iota\|). \tag{A.4}$$

The same argument applies to the $\widetilde{\Pi}_{pp}^\iota$ term. Taking $\mathbb{E}_t[\cdot]$ in Equation (A.3) and dropping the higher-order remainders yields

$$\mathbb{E}_t\Big[\widetilde{\Pi}_{pp}^\iota(s_t)\big(\overline{p}_t^\iota - \widetilde{p}_{t+1}^\iota\big) + \widetilde{\Pi}_{pH}^\iota(s_t)\big(\overline{H}_t^\iota - \widetilde{H}_{t+1}^\iota\big)\Big] = 0. \tag{A.5}$$

The remaining FOC is analogous.

Now define the deviation vector $\mathbf{\Delta x}_t^\iota \equiv \begin{bmatrix} \overline{p}_t^\iota - \mathbb{E}_t[\widetilde{p}_{t+1}^\iota] \\ \overline{H}_t^\iota - \mathbb{E}_t[\widetilde{H}_{t+1}^\iota] \end{bmatrix}$, and the Hessian matrix of $\Pi$ evaluated at $s_t$, $\nabla^2\widetilde{\Pi}^\iota(s_t) \equiv \begin{bmatrix} \widetilde{\Pi}_{pp}^\iota(s_t) & \widetilde{\Pi}_{pH}^\iota(s_t) \\ \widetilde{\Pi}_{Hp}^\iota(s_t) & \widetilde{\Pi}_{HH}^\iota(s_t) \end{bmatrix}$. Stacking the two linearized FOCs gives $\mathbb{E}_t\Big[\nabla^2\widetilde{\Pi}^\iota(s_t)\Big]\,\mathbf{\Delta x}_t^\iota = 0$. If the Hessian matrix $\nabla^2\widetilde{\Pi}^\iota(s_t)$ is full rank, then $\mathbf{\Delta x}_t^\iota = 0$, which completes the proof. ∎

## A.2 Proof of Proposition 1

*Proof.* Define the value gap between choosing dollar-pricing relative to peso-pricing as

$$\Delta\Pi^{\text{USDI}-\text{COPI}} \equiv \mathbb{E}_t\left[\Pi_i(\overline{p}^{\text{USDI}}, \overline{H}^{\text{USDI}}; s_{t+1}) - \Pi_i(\overline{p}^{\text{COPI}}, \overline{H}^{\text{COPI}}; s_{t+1})\right].$$

Take a second-order Taylor expansion of $\Xi$ around the desired allocation. First-order terms vanish by the envelope property for interior solutions. [13] Thus, we compute each second-order terms.

[13] The proof also applies in Section 4, where the state-contingent hedging choice may lie at a corner and hence the hedging FOC need not vanish. Nonetheless, the first-order hedging terms cancel exactly across invoicing regimes because the functional form of $\Pi_i(p, H)$ is the same for every $\iota$.

The pricing-curvature term is given by

$$\begin{aligned}
\text{term}_{pp} &= \frac{1}{2}\widetilde{\Pi}_{pp}\,\mathbb{E}_t\Big[(\overline{p}_t^{\text{USDI}} - \widetilde{p}_{t+1}^{\text{USDI}})^2 - (\overline{p}_t^{*\text{COPI}} - \widetilde{p}_{t+1}^{*\text{COPI}})^2\Big] \\
&= \frac{1}{2}\widetilde{\Pi}_{pp}\,\mathbb{E}_t\Big[\Big((\overline{p}_t^{\text{USDI}} - \widetilde{p}_{t+1}^{\text{USDI}}) + (\overline{p}_t^{*\text{COPI}} - \widetilde{p}_{t+1}^{*\text{COPI}})\Big)\Big((\overline{p}_t^{\text{USDI}} - \widetilde{p}_{t+1}^{\text{USDI}}) - (\overline{p}_t^{*\text{COPI}} - \widetilde{p}_{t+1}^{*\text{COPI}})\Big)\Big] \\
&= \frac{1}{2}\widetilde{\Pi}_{pp}\,\text{cov}_t\Big(\overline{p}_t^{\text{USDI}} + \overline{p}_t^{*\text{COPI}} - \widetilde{p}_{t+1}^{\text{USDI}} - \widetilde{p}_{t+1}^{*\text{COPI}}, (\overline{p}_t^{\text{USDI}} - \overline{p}_t^{*\text{COPI}}) - (\widetilde{p}_{t+1}^{\text{USDI}} - \widetilde{p}_{t+1}^{*\text{COPI}})\Big) \\
&= \frac{1}{2}\widetilde{\Pi}_{pp}\,\text{cov}_t\Big(-\widetilde{p}_{t+1}^{\text{USDI}} - \widetilde{p}_{t+1}^{*\text{COPI}}, -(\widetilde{p}_{t+1}^{\text{USDI}} - \widetilde{p}_{t+1}^{*\text{COPI}})\Big) \\
&= \frac{1}{2}\widetilde{\Pi}_{pp}\,\text{cov}_t\Big(\widetilde{p}_{t+1}^{\text{USDI}} + \widetilde{p}_{t+1}^{*\text{COPI}}, \widetilde{p}_{t+1}^{\text{USDI}} - \widetilde{p}_{t+1}^{*\text{COPI}}\Big) \\
&= -\frac{1}{2}\widetilde{\Pi}_{pp}\,\text{cov}_t\Big(s_{t+1}, 2\widetilde{p}_{t+1}^{\text{USDI}} + s_{t+1}\Big),
\end{aligned}$$

where the third equality uses Lemma 2, so that

$$\mathbb{E}_t\Big[(\overline{p}_t^{\text{USDI}} - \overline{p}_t^{*\text{COPI}}) - (\widetilde{p}_{t+1}^{\text{USDI}} - \widetilde{p}_{t+1}^{*\text{COPI}})\Big] = 0,$$

and the last equality uses the relationship between desired prices across invoicing regimes in Equation (4).

Moreover, the pure hedging-curvature term cancels out:

$$\text{term}_{HH} = \frac{1}{2}\widetilde{\Pi}_{HH}\,\mathbb{E}_t\Big[(\overline{H}_t^{\text{USDI}} - \widetilde{H}_{t+1}^{\text{USDI}})^2 - (\overline{H}_t^{\text{COPI}} - \widetilde{H}_{t+1}^{\text{COPI}})^2\Big] = 0, \tag{A.6}$$

because desired hedging is the same across currency invoicing regimes.

Lastly, the remaining cross-term is

$$\begin{aligned}
\text{term}_{pH} &= \widetilde{\Pi}_{pH}\,\mathbb{E}_t\Big[(\overline{H}_t - \widetilde{H}_{t+1})\Big((\overline{p}_t^{\text{USDI}} - \widetilde{p}_{t+1}^{\text{USDI}}) - (\overline{p}_t^{*\text{COPI}} - \widetilde{p}_{t+1}^{*\text{COPI}})\Big)\Big] \\
&= -\widetilde{\Pi}_{pH}\,\text{cov}_t(\widetilde{H}_{t+1}, s_{t+1}).
\end{aligned} \tag{A.7}$$

Collecting terms gives

$$\begin{aligned}
\Xi &= -\frac{1}{2}\widetilde{\Pi}_{pp}\,\text{cov}_t\Big(s_{t+1}, s_{t+1} + 2\widetilde{p}_{t+1}^{\text{USDI}}\Big) - \widetilde{\Pi}_{pH}\,\text{cov}_t(\widetilde{H}_{t+1}, s_{t+1}) \\
&= -\sigma_s^2\left[\frac{1}{2}\widetilde{\Pi}_{pp}(1+2\beta) + \widetilde{\Pi}_{pH}\beta_H\right],
\end{aligned} \tag{A.8}$$

where exchange-rate betas are defined in Definition 1. Finally, since $\widetilde{\Pi}_{pp}(s_t) < 0$ by assumption, the condition $\Delta\Pi^{\text{USDI}-\text{COPI}} > 0$ is equivalent to Equation (8). ■

## A.3 Proof of Proposition 2

*Proof.* Under the pricing condition in Equation (22), the desired price satisfies

$$\widetilde{p}_{t+1} = \text{const} - (1-\alpha)\log\Big((1-\widetilde{H}_{t+1})S_{t+1} + \widetilde{H}_{t+1}F_t\Big).$$

Hence, evaluated at the desired allocation,

$$\widetilde{X}_{pp}(S_t) = -(\eta-1)\exp[-(\eta-1)\widetilde{p}]\,S_t, \qquad \widetilde{X}_{pH}(S_t) = (\eta-1)\exp[-(\eta-1)\widetilde{p}]\,(1-\alpha)\tau. \tag{A.9}$$

Taking the ratio yields the final results. ■

### Proof of Proposition 2

*Proof.* Because desired hedging is a corner solution in this environment $\widetilde{H}_{t+1} = \mathbf{1}(s_{t+1} < f_t)$. Substituting into $\beta_H$, we obtain

$$\begin{aligned}
\beta_H \equiv \frac{\text{cov}_t(\widetilde{H}_{t+1}, s_{t+1})}{\text{var}(s_{t+1})} &= \mathbb{E}_t[s_{t+1}\mathbf{1}\{s_{t+1} < f_t\}] - \Pr(s_{t+1} < f_t)\mathbb{E}_t[s_{t+1}] \\
&= \Pr(s_{t+1} < f_t)\Big(\mathbb{E}_t[s_{t+1} \mid s_{t+1} < f_t] - \mathbb{E}_t[s_{t+1}]\Big) \\
&= \Phi(z)\left[\left(\mathbb{E}_t[s_{t+1}] - \sigma_s\frac{\phi(z)}{\Phi(z)}\right) - \mathbb{E}_t[s_{t+1}]\right] \\
&= -\sigma_s\phi(z) \\
&= -\frac{1}{\sigma_s}\phi\left(\frac{\tau}{\sigma_s}\right).
\end{aligned} \tag{A.10}$$

where $z = \frac{f_t - \mathbb{E}_t[s_{t+1}]}{\sigma_s}$ and $\phi$ is the standard normal PDF. Next, we compute $\widetilde{\Pi}_{pH}(S_t)$ and $\widetilde{\Pi}_{pp}(S_t)$. First, notice that, for $z \in \{p, H\}$, we have

$$\widetilde{\Pi}_{pz} = \frac{\partial}{\partial z}\left[\frac{\partial\Pi}{\partial X}\frac{\partial X}{\partial p}\right] = \frac{\partial}{\partial z}\left(\frac{\partial\Pi}{\partial X}\right)\frac{\partial X}{\partial p} + \frac{\partial\Pi}{\partial X}\frac{\partial}{\partial z}\left(\frac{\partial X}{\partial p}\right) = \underbrace{\frac{\partial^2\Pi}{\partial X^2}\frac{\partial X}{\partial z}\frac{\partial X}{\partial p}}_{\to 0 \text{ as } p \to \widetilde{p}} + \frac{\partial\Pi}{\partial X}\frac{\partial^2 X}{\partial p\,\partial z} = \widetilde{\Pi}_X\widetilde{X}_{pz}.$$

Thus, we only need to compute $\widetilde{X}_{pH}(S_t)$ and $\widetilde{X}_{pp}(S_t)$. Note that under the pricing condition in Equation (22), the desired price satisfies

$$\widetilde{p}_{t+1} = \text{constant} - (1-\alpha)\log\Big((1-\widetilde{H}_{t+1})S_{t+1} + \widetilde{H}_{t+1}F_t\Big).$$

Hence, evaluated at the desired allocation,

$$\widetilde{X}_{pp}(S_t) = -(\eta-1)\exp[-(\eta-1)\widetilde{p}]\,S_t, \qquad \widetilde{X}_{pH}(S_t) = (\eta-1)\exp[-(\eta-1)\widetilde{p}]\,(1-\alpha)\tau. \tag{A.11}$$

Finally, substituting in, we obtain the closed-form expression

$$\beta^\tau \equiv \frac{\widetilde{\Pi}_{pH}}{\widetilde{\Pi}_{pp}}\beta_H = \frac{\widetilde{X}_{pH}}{\widetilde{X}_{pp}}\beta_H = \left(-\frac{(1-\alpha)\tau}{S_t}\right)\left(-\frac{1}{\sigma_s}\phi\left(\frac{\tau}{\sigma_s}\right)\right) = \frac{1-\alpha}{S_t}\frac{\tau}{\sigma_s}\phi\left(\frac{\tau}{\sigma_s}\right). \tag{A.12}$$

■

## A.4 Proof of Lemma 3

*Proof.* From Equation (22), taking logs and evaluating at the desired allocation gives $\mathrm{d}\widetilde{p}_{i,t+1} = -(1-\alpha_i)\,\mathrm{d}\log\big((1-\widetilde{H}_{i,t+1})S_{t+1} + \widetilde{H}_{i,t+1}F_t\big)$. Because desired hedging is a corner solution, $\widetilde{H}_{i,t+1} =$

$\mathbf{1}(s_{t+1} < f_t)$, that is in logs, $\log\big((1-\widetilde{H}_{i,t+1})S_{t+1} + \widetilde{H}_{i,t+1}F_t\big) = \max(s_{t+1}, f_t)$,. Finally, using $\mathbf{1}(s_{t+1} > f_t) = 1 - \mathbf{1}(s_{t+1} < f_t)$, we obtain

$$\mathrm{d}\widetilde{p}_{i,t+1} = -(1-\alpha_i)\big(1 - \mathbf{1}(s_{t+1} < f_t)\big)\mathrm{d}s_{t+1} = -(1 - \alpha_i - (1-\alpha_i)\mathbf{1}(s_{t+1} < f_t))\,\mathrm{d}s_{t+1}.$$

Thus Equation (23) holds with $\mathring{\alpha}_i = \alpha_i$ and $\mathring{\tau}_i = (1-\alpha_i)\mathbf{1}(s_{t+1} < f_t)$. ∎

## A.5 Proof of Proposition 2

*Proof.* By assumption, $s_{t+1} \mid s_t \sim \mathcal{N}(s_t, \sigma_s^2)$ and $\widetilde{p}_{i,t+1}$ is differentiable almost everywhere in $s_{t+1}$. Hence Stein's lemma implies

$$\beta_p \equiv \frac{\mathrm{cov}_t(\widetilde{p}_{i,t+1}, s_{t+1})}{\mathrm{var}_t(s_{t+1})} = \mathbb{E}_t\left[\frac{\mathrm{d}\widetilde{p}_{i,t+1}}{\mathrm{d}s_{t+1}}\right].$$

To compute the right-hand side, we take expectations of Equation (23), which yields $\overline{\mathring{\tau}}_i \equiv \mathbb{E}_t[\mathring{\tau}_i] = (1-\alpha_i)\mathbb{P}_t(s_{t+1} < f_t) = (1-\alpha_i)\Phi\left(-\frac{\tau}{\sigma_s}\right)$. Next, from the proof of Proposition 2, we have $\beta^\tau = (1-\alpha_i)\frac{\tau}{\sigma_s}\phi\left(\frac{\tau}{\sigma_s}\right)$, where we have used the normalization $S_t = 1$. ∎

## A.6 Proof of Corollary 2

*Proof.* It suffices to study the function $g(x) = x\phi(x)$ for $x \geq 0$, where $\phi(\cdot)$ is the standard normal PDF. Clearly, $g(0) = 0$. Moreover, $g(x) = \frac{x}{\sqrt{2\pi}}e^{-x^2/2}$, so as $x \to \infty$, the exponential term dominates the linear term $x$, implying $\lim_{x\to\infty} g(x) = \lim_{x\to\infty} \frac{x}{\sqrt{2\pi}}e^{-x^2/2} = 0$. Hence $g(0) = 0$ and $g(\infty) = 0$, which completes the proof. ∎

## A.7 Proof of Corollary 3

*Proof.* Notice that $\beta_p$ increases with $\overline{\mathring{\tau}}_i$ and therefore decreases with $\tau$. It therefore remains to show that $g(x)$ in Appendix Section A.6 is decreasing for $x > 1$, where $x \equiv \tau/\sigma_s$. Since $g'(x) = \phi(x)(1 - x^2) < 0$ for $x > 1$, the claim follows. ∎

## A.8 Illustrative Example of FX Forward Contract

To hedge, firms expecting USD receivables tomorrow take a short position in USD forwards, whereas firms expecting USD payables take a long position in USD forwards.

Table A.1 illustrates that FX forwards hedge the sign of a firm's net dollar cash-flow exposure. An exporter expecting a dollar receivable is effectively *long USD*: when the dollar appreciates ($S_T \uparrow$), its peso revenue rises, and when the dollar depreciates, its peso revenue falls. To hedge this exposure, the exporter takes a *short USD forward* position, whose payoff $(F^{\text{bid}} - S_T)N$ moves in the opposite direction of the unhedged cash flow. Conversely, an importer expecting a dollar payable is effectively *short USD*: when the dollar appreciates, its peso cost rises. To hedge this exposure, the importer takes a *long USD forward* position, whose payoff $-(F^{\text{ask}} - S_T)N$ again offsets the exchange-rate movement in operating cash flow. In both cases, the hedge converts the state-contingent peso cash flow $\pm S_T N$ into the fixed peso cash flow $\pm FN$. Thus, exporters short USD forward while importers long USD

Table A.1: Illustrative Example of Firm Hedging with FX Forward

| **Set up** | Exchange Rate: $S_0 = 100$ Peso/USD; Quantity of Goods $N = 1$ | |
|---|---|---|
| | **Exporter (USD receivable)** | **Importer (USD payable)** |
| **Exposure** | Long USD exposure ($S_T \uparrow$, peso revenue $\uparrow$) | Short USD exposure ($S_T \uparrow$, peso cost $\uparrow$) |
| **Hedge with forward** | Short USD forward | Long USD forward |
| **Forward price** | $F^{\text{bid}} = 101$ | $F^{\text{ask}} = 99$ |
| **Forward payoff vs. unhedged** | $(F^{\text{bid}} - S_T)N$ | $-(F^{\text{ask}} - S_T)N$ |
| **Scenario A: USD appreciates** ($S_T = 110$) | | |
| Unhedged net peso cash flow | $+S_T N = +110$ | $-S_T N = -110$ |
| Forward payoff (increment) | $(101 - 110)N = -9$ | $-(99 - 110)N = +11$ |
| Hedged net peso cash flow | $+F^{\text{bid}} N = +101$ | $-F^{\text{ask}} N = -99$ |
| **Scenario B: USD depreciates** ($S_T = 95$) | | |
| Unhedged net peso cash flow | $+S_T N = +95$ | $-S_T N = -95$ |
| Forward payoff (increment) | $(101 - 95)N = +6$ | $-(99 - 95)N = -4$ |
| Hedged net peso cash flow | $+F^{\text{bid}} N = +101$ | $-F^{\text{ask}} N = -99$ |

forward because the forward position must have the opposite sign of the firm's underlying dollar exposure.